

The combined Influence of Instantons and Strong Magnetic Fields on Quark Matter

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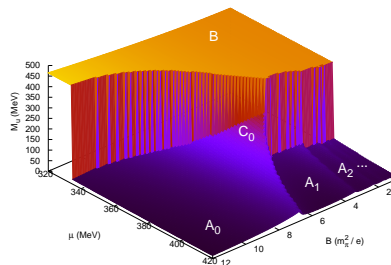
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Outline

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Spontaneous CP violation at $\theta = \pi$ I

- ▶ Kharzeev, Pisarski and Tytgat (1998) suggested using χ^{PT} that at high temperature metastable CP-violating states could exist
- ▶ Dashen (1971): spontaneous CP violation (SCPV) at $\theta = \pi$
- ▶ It has been studied extensively in Chiral Perturbation Theory, Witten (1980), di Vecchia & Veneziano (1980), Smilga (1999), Tytgat (2000), Akemann *et al.* (2002), Creutz (2004), Metlitski & Zhitnitsky (2005, 2006), ...

Spontaneous CP violation at $\theta = \pi$ II

- ▶ We look at the NJL-model
Nambu & Jona-Lasinio (1961)
- ▶ The effect of instantons are mimicked by the 't Hooft determinant interaction
't Hooft (1976, 1986)
- ▶ In this part of the talk the focus is on the current mass and temperature dependence

Arguments of Kharzeev, Pisarski and Tytgat

- ▶ The three-flavor chiral Lagrangian

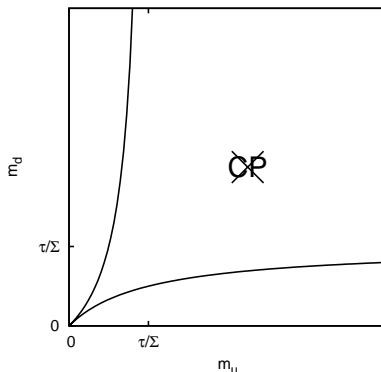
$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} \left[\partial_\mu U \partial^\mu U^\dagger \right] + \Sigma \text{Re} [\text{Tr}(MU)] - \frac{\tau}{2} (i \log \det U + \theta)^2$$

where $\Sigma = |\langle \bar{\psi} \psi \rangle|$, M the quark mass matrix, τ the topological susceptibility and U an $U(3)$ field representing the pseudo-Goldstone bosons

- ▶ Potential contained in this Lagrangian has CP violating metastable states when $\tau/\Sigma < 0.251 m_u$ (using the values of Tytgat (2000))
- ▶ Kharzeev, Pisarski and Tytgat considered the option that τ/Σ is temperature dependent, i.e. $\tau/\Sigma \sim (T_c - T)^{3/2}$
- ▶ Close to T_c CP violating metastable states appear

Spontaneous CP violating at $\theta = \pi$

Using the results of Tytgat (2000) the following phase diagram can be obtained



- ▶ Asymptotes depend on ratio τ/Σ
- ▶ Study same diagram in the NJL model

The NJL model

The Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}}$$

with

Free part $\mathcal{L}_0 = \bar{\psi} (i\partial - m + \gamma_0 \mu) \psi$

Chiral symmetric interaction $\mathcal{L}_{\text{sym}} = G_1 \left[(\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} \tau_a i \gamma_5 \psi)^2 \right]$

Determinant interaction $\mathcal{L}_{\text{det}} = 2G_2 e^{i\theta} [\det \{ \bar{\psi} (1 - \gamma_5) \psi \} + \text{h.c.}]$

and

$$m = \text{diag} (m_u, m_d),$$

We want to vary G_2

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix},$$

Usually $G_1 = G_2$

The τ_a are the generators of $U(2)$

Parameters are fitted to experimental values of m_π , f_π and $\langle \bar{\psi} \psi \rangle$

The effective potential in mean-field approximation

- ▶ The interaction terms are “linearized”

$$(\bar{\psi}\tau_a\psi)^2 \simeq 2 \langle \bar{\psi}\tau_a\psi \rangle \bar{\psi}\tau_a\psi - \langle \bar{\psi}\tau_a\psi \rangle^2$$

- ▶ Consider only charge neutral condensates $\langle \bar{\psi}\tau_0\psi \rangle$, $\langle \bar{\psi}\tau_3\psi \rangle$, $\langle \bar{\psi}\tau_0 i\gamma_5\psi \rangle$, and $\langle \bar{\psi}\tau_3 i\gamma_5\psi \rangle$
- ▶ Lagrangian quadratic in the quark fields
- ▶ Perform the integration over the quark fields

The effective potential

The thermal effective potential at $\theta = \pi$ is

$$\mathcal{V} = \frac{\alpha_0^2 + \beta_3}{4(G_1 - G_2)} + \frac{\alpha_3^2 + \beta_0^2}{4(G_1 + G_2)} - TN_c \sum_{p_0=(2n+1)\pi T} \int \frac{d^3 p}{(2\pi)^3} \ln \det [i\gamma_0 p_0 + \gamma_i p_i - \mathcal{M} - \gamma_0 \mu]$$

where

$$\begin{aligned} \alpha_0 &= -2(G_1 - G_2) \langle \bar{\psi} \tau_0 \psi \rangle & \alpha_3 &= -2(G_1 + G_2) \langle \bar{\psi} \tau_3 \psi \rangle \\ \beta_0 &= -2(G_1 + G_2) \langle \bar{\psi} \tau_0 i\gamma_5 \psi \rangle & \beta_3 &= -2(G_1 - G_2) \langle \bar{\psi} \tau_3 i\gamma_5 \psi \rangle \end{aligned}$$

and

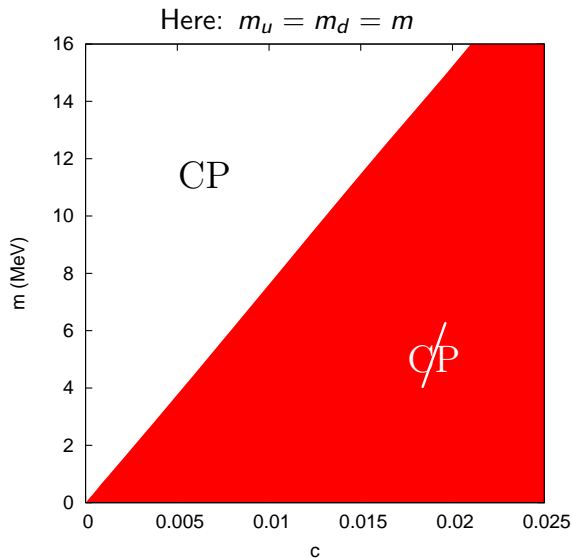
$$\mathcal{M} = \left(\frac{m_u + m_d}{2} + \alpha_0 + \beta_0 i\gamma_5 \right) \tau_0 + \left(\frac{m_u - m_d}{2} + \alpha_3 + \beta_3 i\gamma_5 \right) \tau_3$$

We take $G_1 \neq G_2$

Effect of the instanton interaction

- ▶ Keep $G_1 + G_2$ fixed, determines $\theta = 0$ -physics
- ▶ Vary $c = G_2/(G_1 + G_2)$ between 0 and 1/2
Frank, Buballa & Oertel (2003)
- ▶ Exact value of c is unknown in Nature, from $N_f = 3$ and m_η considerations probably $c \approx 0.2$
Frank, Buballa & Oertel (2003)

The (c, m) phase diagram



For every m a critical c exists, i.e., the instanton interaction has to be strong enough w.r.t. m for SCPV to occur.

The (m_u, m_d) -phase-diagram for $\theta = \pi$ at $c = .4, T = 0$

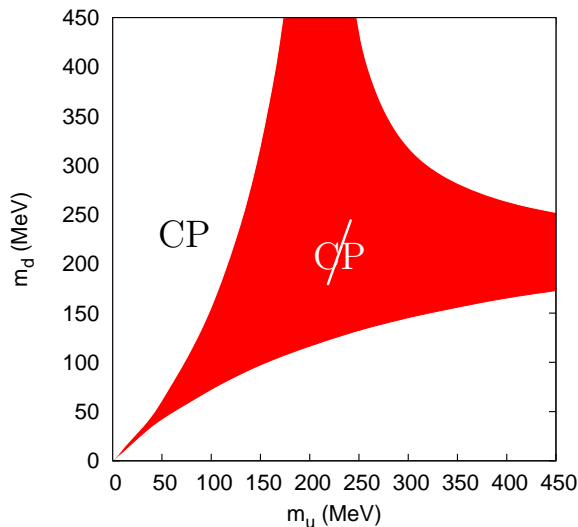


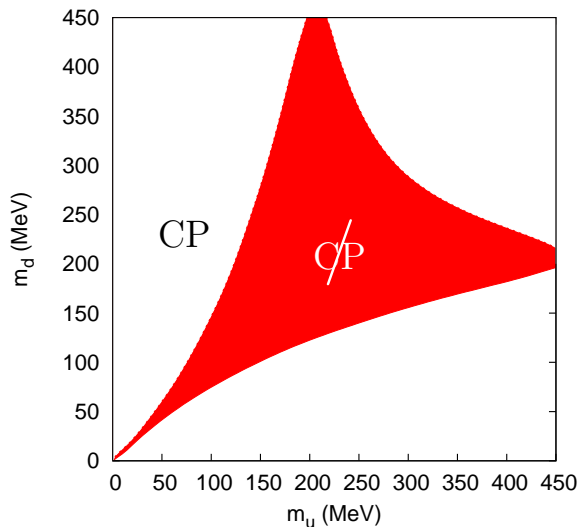
Figure similar to 3 flavor χ PT, Creutz (2004)

Two flavor NJL asymptotes depend on c instead of m_s

In two flavor χ PT asymptotes depend on τ/Σ , no upper bound present

Tytgat (2000)

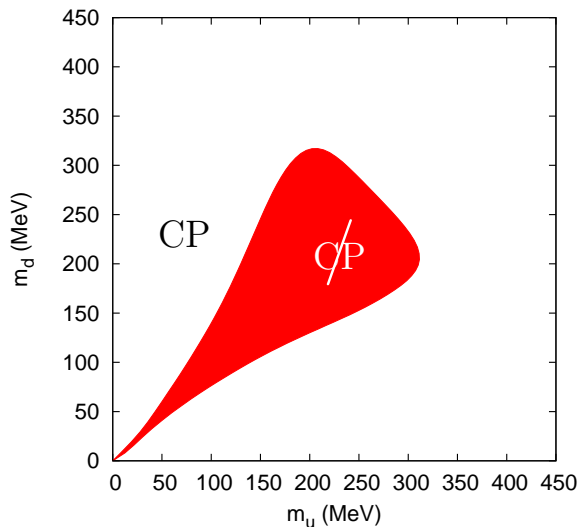
The (m_u, m_d) -phase-diagram for $\theta = \pi$ at $c = .4, T = 120$



The CP-violating region becomes smaller with increasing temperature

SCPV is a low-energy phenomenon

The (m_u, m_d) -phase-diagram for $\theta = \pi$ at $c = .4$, $T = 150$



Mainly the high-mass regime is affected

The asymptotes are not much affected

This may indicate that suggestions for metastable states may not hold in QCD

Conclusions

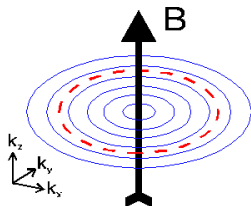
- ▶ Instantons can cause spontaneous SCPV at $\theta = \pi$
- ▶ The actual occurrence of SPCV depends on the strength of the instanton interaction w.r.t. quark masses
- ▶ SCPV is a low-energy phenomenon
- ▶ No metastable phases were found neither CP violating nor CP conserving ones

Part 2: Strong magnetic fields

- ▶ Very strong magnetic fields occur in:
 - ▶ Heavy ion collisions: 10^{19} G
Kharzeev, McLerran & Warringa (2009); Skokov, Illarionov & Toneev (2009)
 - ▶ Ordinary neutron stars up to 10^{13} G, magnetars up to 10^{15} G and possibly in core up to 10^{18} G
Duncan & Thompson (1992); Thompson & Duncan (1993); Lattimer & Prakash (2007)
- ▶ In order to understand these systems we have to understand how magnetic fields affect quark matter

Charged particles in a strong magnetic field I

- ▶ Effect: Landau quantization
Landau & Lifshitz (1977)



- ▶ Discrete levels get subsequently filled \rightarrow oscillation with B
- ▶ de Haas-van Alphen effect
- ▶ Similar effects on quark matter?

Charged particles in a strong magnetic field II

- ▶ Perform study using two-flavor NJL model
- ▶ This part: combined study of the influence of instantons and magnetic fields at $\theta = 0$
- ▶ We will assume that no CP violation takes place ($\beta_0 = \beta_3 = 0$)
- ▶ For convenience we introduce the constituent masses

$$\begin{aligned}M_u &= m + \alpha_0 + \alpha_3 \\&= m - 2(G_1 + G_2) \langle \bar{\psi} \tau_0 \psi \rangle - 2(G_1 - G_2) \langle \bar{\psi} \tau_3 \psi \rangle \\M_d &= m + \alpha_0 - \alpha_3 \\&= m - 2(G_1 + G_2) \langle \bar{\psi} \tau_0 \psi \rangle + 2(G_1 - G_2) \langle \bar{\psi} \tau_3 \psi \rangle\end{aligned}$$

Including the magnetic field

- ▶ Choose magnetic field in z-direction
- ▶ Choose gauge such that $A^\mu = (0, -By, 0, 0)$
- ▶ Obtain new dispersion relation

$$p_{0n}^2 = p_z^2 + M^2 + (2n + 1 - \sigma)|q|B.$$

- ▶ Thermal integral transforms according to
Chakrabarty (1996); Fraga & Mizher (2008); Fukushima & Warringa (2008)

$$T \sum_{p_0} \int \frac{d^3 p}{(2\pi)^3} \rightarrow \frac{|q|BT}{2\pi} \sum_{p_0} \sum_{n=0}^{\infty} \int \frac{dp_z}{2\pi},$$

The effective potential at nonzero magnetic field

$$\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_1(B) + \mathcal{V}_2(B, \mu, T)$$

with

$$\mathcal{V}_0 = \frac{(M_0 - m)^2}{4(G_1 + G_2)} + \frac{M_3^2}{4(G_1 - G_2)} - 2N_c \sum_{f=u}^d \int \frac{d^3p}{(2\pi)^3} \sqrt{\mathbf{p}^2 + M_f^2},$$

$$\mathcal{V}_1(B) = -\frac{N_c}{2\pi^2} \sum_{f=u}^d (|q_f|B)^2 \left[\zeta'(-1, x_f) - \frac{1}{2}(x_f^2 - x_f) \ln x_f + \frac{x_f^2}{4} \right],$$

$$\begin{aligned} \mathcal{V}_2(B, \mu T) = & -\frac{N_c}{2\pi} \sum_{\sigma, n, f} (|q_f|B) \int \frac{dp_z}{2\pi} \left\{ T \ln \left[1 + e^{-[E_p(B) + \mu_f]/T} \right] \right. \\ & \left. + T \ln \left[1 + e^{-[E_p(B) - \mu_f]/T} \right] \right\}, \end{aligned}$$

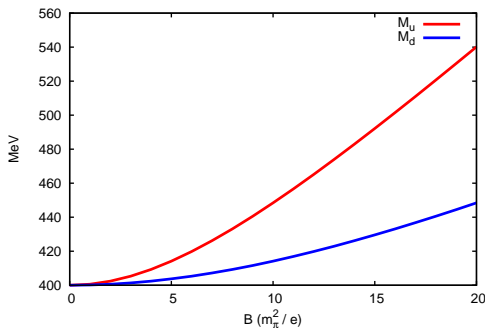
where $x_f = \frac{M_f^2}{2|q_f|B}$ and $\zeta'(-1, x_f) = \frac{d\zeta(z, x_f)}{dz} \big|_{z=-1}$

B -dependence of masses without instanton interaction

- ▶ B field anti-aligns helicities of quarks and antiquarks \rightarrow more strongly bound by interaction

Klevansky (1992)

- ▶ Constituent quark masses increase with magnetic field
($m_\pi^2/e = 0.33 \times 10^{19}$ G)

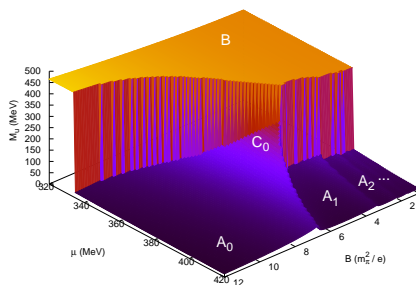


Magnetic enhancement of χ^{SB}

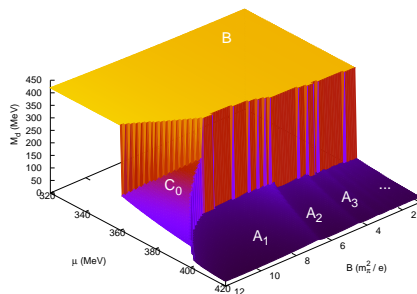
small isospin violation

Nonzero chemical potential at $G_2 = 0$

Up quark



Down quark



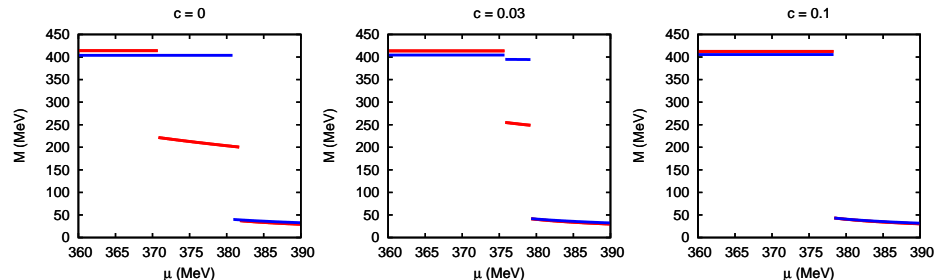
- ▶ Similar phase structure found by Ebert et al. 2000; Ebert and Klimenko (2003) with $G_2 = G_1$
- ▶ Discontinuous de Haas-van Alphen effect
- ▶ A region exists with a considerable mass difference \rightarrow large spontaneous breaking of isospin ($\langle \bar{\psi} \tau_3 \psi \rangle \neq 0$)

Effect of the instanton interaction I

- ▶ Strong magnetic field induces different behavior quarks
- ▶ Spontaneous isospin violation
- ▶ Interaction is flavor mixing \rightarrow counters effect of magnetic field
- ▶ Investigate this competition as function of c

Effect of the instanton interaction II

$$B = 5m_\pi^2/e$$

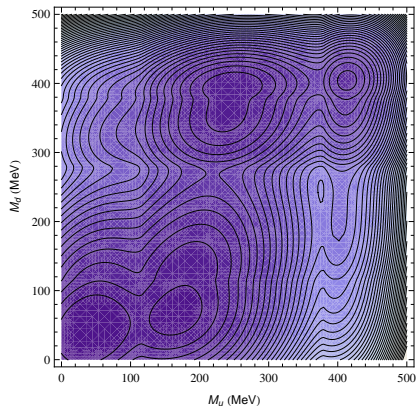


- ▶ C_0 phase disappears with increasing c ($= G_2/(G_1 + G_2)$)
- ▶ Regions with large mass differences disappear
- ▶ Behavior similar to Frank, Buballa & Oertel (2003)
($\mu_I \neq 0, B = 0$)

Metastable states

Effective potential as function of M_u and M_d at

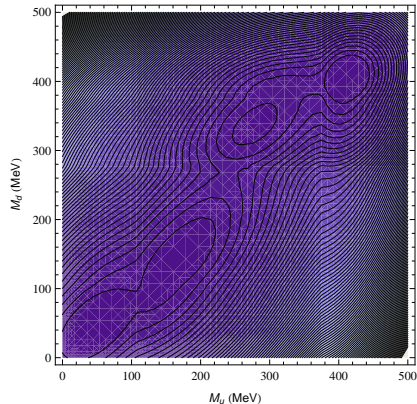
$$B = 5m_\pi^2/e, \quad c = 0.03, \quad \mu = 378 \text{ MeV}$$



Metastable states

Effective potential as function of M_u and M_d at

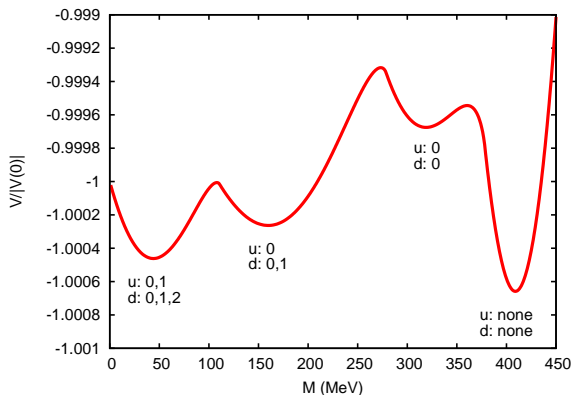
$$B = 5m_\pi^2/e, \quad c = 0.1, \quad \mu = 378 \text{ MeV}$$



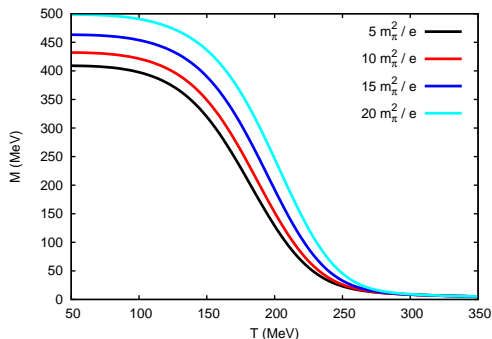
Nearly degenerate metastable states

Normalized effective potential as function of $M = M_u = M_d$ at

$$B = 5m_\pi^2/e, \quad c = 1/2 \quad (G_1 = G_2), \quad \mu = 378 \text{ MeV}$$



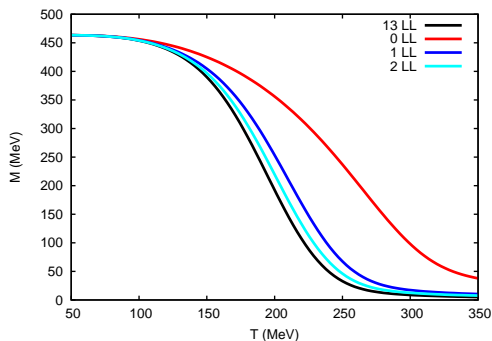
T -dependence of masses at $c = 1/2$



- ▶ High T χ symmetry restoring transition remains crossover ($m_q \neq 0$) at nonzero B
- ▶ First order transition in linear sigma model coupled to quarks
Stronger at larger B

Mizher & Fraga (2009)

Influence of higher Landau levels $c = 1/2$, $B = 15m_\pi^2/e$



- ▶ More Landau levels \rightarrow transition sharper
- ▶ Qualitative details unchanged

Conclusions

- ▶ A magnetic field enhances chiral symmetry breaking and allows for the possibility of spontaneous isospin breaking
- ▶ The instanton interaction counters the effect of magnetic field
- ▶ de Haas-van Alphen effect is discontinuous in NJL model
- ▶ Also metastable states develop that differ considerably in the amount of chiral symmetry breaking
- ▶ High-temperature phase transition remains crossover at nonzero B
- ▶ NJL: $G_1 \neq G_2$ gives important qualitative differences
 - ▶ $G_1 = G_2 \rightarrow M_u = M_d \rightarrow$ no violation of isospin

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